Aufgabe 7 $V = \mathbb{R}^{2\times 1}$ affin

$$\lambda : \mathbb{R}^{2\times 1} \to \mathbb{R} : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto x_1^2 - 2x_1x_2 + 2x_2^2 - 2x_1 + 7 \qquad \text{qu. Abb.}$$

a) ges. qu. Form $q : \mathbb{R}^{3\times 1} \to \mathbb{R}$

$$q : \mathbb{R}^{3\times 1} \to \mathbb{R} : \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} \mapsto x_1^2 - 2x_1x_2 + 2x_2^2 - 2x_0x_1 + 7x_0^2$$

b) Matrix von q bzgl

$$\begin{array}{c|cc} 7 & -1 & 0 \\ \hline -1 & 1 & -1 \\ 0 & -1 & 2 \\ \hline 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \mapsto \begin{array}{ccc} 6 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \\ \hline 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \mapsto \begin{array}{ccc} 6 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \\ \hline \end{array}$$

$$\mapsto \begin{array}{ccc} 6 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \\ \hline 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{array} \mapsto \begin{array}{ccc} 6 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ \hline \end{array}$$

$$\xmapsto{s} \xmapsto{z} \begin{array}{c|cc} 5 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 0 & 0 \\ \hline 2 & 1 & 1 \\ 1 & 0 & 1 \end{array}$$

$$\Rightarrow \tilde{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \tilde{b}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \tilde{b}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \text{Koo.}: \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right)$$

c)

$$\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} \mapsto \tilde{x}_1^2 + \tilde{x}_2^2 + 5$$

d) $\Phi(\lambda) = \left\{ \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; \tilde{x}_1^2 + \tilde{x}_2^2 = -5 \right\} = \emptyset$ ~~⇒ Φ(λ) ist Ellipse ⇒ ∃ Mittelpunkt~~

Aufgabe 8

$$q: \mathbb{R}^{4\times 1} \to \mathbb{R}: \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto x_0^2 + 4x_0x_3 + x_1^2 - x_2^2 - 4x_2x_3$$

a) ges.: Gl. v. $\Psi(q)$ in proj. Normalform, Typ?

$$\begin{matrix} & & & \cdot -2 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 \\ 2 & 0 & -2 & 0 \end{matrix} \mapsto \begin{matrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 2 & 0 & -2 & 4 \end{matrix} \;\cdot -2 \mapsto \begin{matrix} & \cdot -\frac{1}{2} & & \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 2 & 0 & 0 & 4 \end{matrix}$$

$$\cancel{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \end{matrix}}$$

$$\mapsto \begin{matrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 \end{matrix} \;\cdot -\tfrac{1}{2} \mapsto \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 \end{matrix} \;\cdot -\tfrac{1}{2} \quad \cdot -\tfrac{1}{2}$$

$$\overset{S}{\mapsto}\overset{Z}{\mapsto} \begin{matrix} \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ & 0 & 0 & 0 & 0 \end{matrix}$$

$\Rightarrow$ Gl.: $x_0^2 + x_1^2 - x_2^2 = 0$

Typ: Kegel

b)

$$\begin{array}{cccc|c} x_0 & x_1 & x_2 & x_3 & \\ 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 \\ 2 & 0 & -2 & 0 & 0 \end{array} \;\cdot -2 \qquad \begin{array}{cccc|c} 1 & 0 & 0 & 2 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & -1 & -2 & 0 \\ 0 & 0 & -2 & -4 & \end{array} \;\cdot -2$$

$$\begin{array}{cccc|c} 1 & 0 & 0 & 2 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & +1 & +2 & 0 \\ 0 & 0 & 0 & 0 & \end{array} \qquad \Rightarrow S = \mathbb{R}\begin{pmatrix} -2 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$

c)

$$(0,0,0,1)\begin{pmatrix}1&0&0&2\\0&1&0&0\\0&0&-1&-2\\2&0&-2&0\end{pmatrix} = (2,0,-2,0)$$

$\Rightarrow$ Gl.: $2x_0 - 2x_2 = 0$

9) $(\mathbb{R}^{3\times1},\cdot)$

$$B = (b_1, b_2, b_3) = \begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix} \qquad x = \begin{pmatrix}1\\2\\3\end{pmatrix}$$

~~$b_1 =$~~

a) ges.: $\hat{B}$

muss gelten $\hat{b}_i \cdot b_j = \delta_{ij} \Leftrightarrow \hat{B} \cdot B = E_3 \Leftrightarrow (\hat{B})^T \cdot B = E_3 \Leftrightarrow \hat{B} = (B^{-1})^T$

$$\Rightarrow \begin{array}{ccc}1&1&1\\0&1&1\\0&0&1\\\hline 1&0&0\\0&1&0\\0&0&1\end{array} \rightarrow \begin{array}{ccc}1&0&0\\0&1&0\\0&0&1\\\hline 1&-1&0\\0&1&-0\\0&0&1\end{array} \longmapsto \begin{array}{ccc}1&0&0\\0&1&0\\0&0&1\\\hline 1&-1&0\\0&1&-1\\0&0&1\end{array}\begin{array}{l}\\\\\\ - \hat{b}_1\\ - \hat{b}_2\\ - \hat{b}_3\end{array}$$

~~$\hat{b}_1 \quad \hat{b}_2 \quad \hat{b}_3$~~

b) ~~$\langle B^*, x\rangle = \hat{B} \cdot x = \begin{pmatrix}1&-1&0\\0&1&-1\\0&0&1\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\-1\\3\end{pmatrix}$~~

~~$\langle \hat{B}^*, x\rangle = B \cdot x = (\hat{B})^T \cdot E_3 \cdot x = \begin{pmatrix}1&0&0\\-1&1&0\\0&-1&1\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$~~

$$\langle B^*, x\rangle = \hat{B} \cdot x = (\hat{B})^T \underset{\uparrow}{\cdot} x = \begin{pmatrix}1&-1&0\\0&1&-1\\0&0&1\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\-1\\3\end{pmatrix}$$

$$\langle \hat{B}^*, x\rangle = B \cdot x = B^T \cdot x = \begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1\\3\\6\end{pmatrix}$$

c) $x \circ x = x^T \cdot x = x^T \cdot \underbrace{\hat{B}^T \cdot B}_{=\hat{B} \circ B = E_n} \cdot x = (\hat{B} \cdot x)^T \cdot B \cdot x$

$= \langle \hat{B}, x \rangle \circ \langle B, x \rangle$

$= (-1, -1, 3) \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} = 14$

d) a Gradient von a^* $\Leftrightarrow$ $\langle a^*, x \rangle = a \circ x \quad \forall x \in \mathbb{R}^{3\times 1}$

$\langle b_1^*, x \rangle = \hat{b}_1 \circ x$

$\langle b_2^*, x \rangle = \hat{b}_2 \circ x$

$\langle (b_1^* - b_2^*), x \rangle = \overbrace{b_1^* - b_2^*} \circ x = (\hat{b}_1 - \hat{b}_2) \circ x = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \circ x$

$\left(\langle b_1^*, x \rangle - \langle b_2^*, x \rangle = \hat{b}_1 \circ x - \hat{b}_2 \circ x = (\hat{b}_1 - \hat{b}_2) \circ x \right)$

Aufgabe 10

$(\mathbb{R}^{4\times 1}, \circ)$

$$U = \left[\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right]$$

a) Gl. v. $U^\perp$

$$\cancel{A} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \\ 0 & -1 \end{pmatrix} \xrightarrow{\cdot -2} \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} \\ \begin{matrix} 1 & -1 \\ 0 & -1 \end{matrix} \end{pmatrix}$$

$$\Rightarrow U^\circ \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \Rightarrow \text{LGS:} \quad \begin{aligned} x_1 - x_2 - x_3 &= 0 \\ x_2 + x_4 &= 0 \end{aligned}$$

b)

$$(b_1, b_2) = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$$

c) $\tilde{a}_1 = b_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

$$\tilde{a}_2 = b_1 - \frac{a_1 \circ b_1}{a_1 \circ a_1} a_1 = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 0 \end{pmatrix} - \frac{-1}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1/2 \\ -1 \\ 1/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ -1 \\ -2 \\ 1 \end{pmatrix}.$$

$\tilde{a}_1 \perp \tilde{a}_2$

$\|\tilde{a}_1\| = \sqrt{2}$

$\|\tilde{a}_2\| = \sqrt{10} = \sqrt{2} \cdot \sqrt{5}$

$$\Rightarrow (a_1, a_2) = \cancel{\sqrt{2}\begin{pmatrix} \frac{0}{\sqrt{2}} \\ \frac{0}{\sqrt{2}} \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$$

$$\frac{1}{\sqrt{2}} \left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \\ -2 \\ 1 \end{pmatrix} \right)$$

Aufgabe 11

$(\mathbb{C}^{3\times 1}, \circ)$

$$b_1 = \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix} \quad b_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

a) $a_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad a_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$a_3 = b_1 - \frac{a_2 \circ b_1}{a_2 \circ a_2} a_2 = \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix} - \frac{i}{1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\cancel{\{(0,0,1), \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix}\}}$$

b) bereits normiert

Aufgabe 12

$(\mathbb{C}^{3\times 1}, \circ)$

$$b_1 = \begin{pmatrix} 0 \\ 1 \\ 2i \end{pmatrix} \qquad b_2 = \begin{pmatrix} 1 \\ i \\ 3 \end{pmatrix} \qquad U = (b_1, b_2)$$

a) $\tilde{a}_1 = b_1 = \begin{pmatrix} 0 \\ 1 \\ 2i \end{pmatrix}$

$$\tilde{a}_2 = b_2 - \frac{\tilde{a}_1 \circ b_2}{\tilde{a}_1 \circ \tilde{a}_1} \tilde{a}_1$$

$$= \begin{pmatrix} 1 \\ i \\ 3 \end{pmatrix} + \frac{\cancel{5}i}{\cancel{5}} \begin{pmatrix} 0 \\ 1 \\ 2i \end{pmatrix} = \begin{pmatrix} 1 \\ 2i \\ 1 \end{pmatrix}$$

$\Rightarrow \tilde{a}_1 \perp \tilde{a}_2$

$\|\tilde{a}_1\| = 5$

$\|\tilde{a}_1\| = 6$

$$\Rightarrow (a_1, a_2) = \left(\frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 1 \\ 2i \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2i \\ 1 \end{pmatrix} \right)$$

b) $pr_1 : \mathbb{C}^{3\times 1} \to U : \; x = \cancel{x}_1 + \cancel{x}_2 \mapsto x_1$

$$pr_1 : x \mapsto \frac{a_1 \circ x}{a_1 \circ a_1} \cdot a_1 + \frac{a_2 \circ x}{a_2 \cdot a_2} \cdot a_2$$

~~$\langle E^*, pr(E) \rangle = U \cdot \overline{U}^T =$~~

$$\langle E^*, pr_1(E) \rangle = a_1 \circ \overline{a_1}^T + a_2 \cdot \overline{a_2}^T$$

$$= \frac{1}{5} \begin{pmatrix} 0 \\ 1 \\ 2i \end{pmatrix} (0, 1, -2i) + \frac{1}{6} \begin{pmatrix} 1 \\ 2i \\ 1 \end{pmatrix} (1, -2i, 1)$$

$$= \frac{1}{5} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -2i \\ 0 & 2i & 4 \end{pmatrix} + \frac{1}{6} \begin{pmatrix} 1 & -2i & 1 \\ 2i & 4 & 2i \\ 1 & -2i & 1 \end{pmatrix}$$

$$= \frac{1}{30}\begin{pmatrix} 5 & -10i & 5 \\ 10i & 26 & -2i \\ 5 & 2i & 44 \end{pmatrix}$$

$$\langle E^*, p_1(E)\rangle = E_3 - \langle E^*, p_2(E)\rangle = \ldots$$

Aufgabe 13

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad EW: \lambda_1 = 1, \ \lambda_2 = -1, \ \lambda_3 = 2$$

a) ges.: $P \in O_3$ mit $P^{-1} A P$ Diag-Matrix.

EV zu λ_1:

$$\left.\begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{matrix}\right| 0 \quad \Rightarrow \tilde{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

EV zu λ_2

$$\left.\begin{matrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{matrix}\right| 0 \quad \Rightarrow \tilde{v}_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

EV zu λ_3:

$$\left.\begin{matrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{matrix}\right| 0 \quad \Rightarrow \tilde{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\|\tilde{v}_1\| = \sqrt{2}$$
$$\|\tilde{v}_2\| = \sqrt{6} \qquad \Rightarrow P = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix}$$
$$\|\tilde{v}_3\| = \sqrt{3}$$

b) P^{-1}: $\quad P \in O_3 \Rightarrow P^{-1} = \omega(P^T) = P^T = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix}$

$$P^{-1} \cdot A \cdot P = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ -1 & -1 & 2 \\ 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} \end{pmatrix} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix} \cdot \frac{1}{\sqrt{6}}$$

$$= \frac{1}{6} \begin{pmatrix} 6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & 12 \end{pmatrix} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 2 \end{pmatrix}$$

c) sonst wär P nicht $\in O_3$

Aufgabe 14

$(\mathbb{R}^{3\times1}, \iota)$

$$\langle E^*, f(E)\rangle = \frac{1}{7}\begin{pmatrix}3 & -2 & 6\\ 6 & 3 & -2\\ -2 & 6 & 3\end{pmatrix} \in SO_2$$

a)

$$\begin{array}{ccc|c} 2 & -2 & 6 & \\ 6 & 2 & -2 & 0 \\ -2 & 6 & 2 & \end{array} \quad \cdot(-3)$$

$$\begin{array}{ccc} 2 & -2 & 6 \\ 0 & 8 & -20 \\ 0 & \cancel{2}1 & \cancel{8}2 \end{array} \quad \cdot(-2)$$

$$\begin{array}{ccc} 2 & 0 & 10 \\ 0 & 0 & -36 \\ 0 & 1 & 2 \end{array}$$

$$\begin{array}{ccc|c} -4 & -2 & 6 & \\ 6 & -4 & -2 & 0 \\ -2 & 6 & -4 & \end{array} \quad \cdot(-2) \quad \cdot 3$$

$$\begin{array}{ccc|c} 0 & -14 & 14 & \\ 0 & 14 & -14 & 0 \\ -1 & 3 & -2 & \end{array}$$

$$\Rightarrow \begin{pmatrix}1\\1\\1\end{pmatrix}$$

b) $\cos\varphi = \frac{\operatorname{spur} f - 1}{2} = \left(3\cdot\frac{3}{7} - 1\right)\cdot\frac{1}{2} = +\frac{2}{14} = +\frac{1}{7}$

c) $\begin{pmatrix}1 & 0 & 0\\ 0 & \frac{1}{7} & -\alpha\\ 0 & \alpha & \frac{1}{7}\end{pmatrix}$ $\quad \alpha = \sin\left(\arccos\frac{1}{7}\right)$

d) $e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad f(e_1) = \frac{1}{7}\begin{pmatrix}3\\6\\-2\end{pmatrix}$

$$\cos\sphericalangle(e_1, f(e_1)) = \frac{e_1 \circ f(e_1)}{\|e_1\|\cdot\|f(e_1)\|} = \frac{\frac{3}{7}}{1} = \underline{\underline{\frac{3}{7}}}$$

Aufgabe 15

$f \in L(\mathbb{R}^{3\times1}, \mathbb{R}^{2\times1})$

$\langle E^*, f(E) \rangle = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

a) $\langle E^*, (\hat{f} \circ f)(E) \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

$$\chi_{\hat{f} \circ f} = \begin{vmatrix} 1-x & 0 & 1 \\ 0 & 1-x & 1 \\ 1 & 1 & 2-x \end{vmatrix} = (1-x)^2(2-x) - 2(1-x)$$

$$= (1-x)\left((1-x)(2-x) - 2\right) \cancel{(2-x-1)}$$

$$\cancel{= (1-x)^3 \Rightarrow EW: \lambda = 1}$$

$$= (1-x)(-3x + x^2) = x(1-x)(3-x)$$

EW: $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 3$

EV: λ_2

$$\begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{matrix} \Rightarrow \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

EV: λ_1

$$\begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{matrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

EV: λ_3:

$$\begin{matrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & -1 \end{matrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow B = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right)$$

b) $\omega_1 = 1$, $\omega_2 = \sqrt{3}$

Aufgabe 18

$$A = \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 6 & 8 \\ 8 & 11 \end{pmatrix}$$

a) $\det B_{(1)} = 6$

$\det B_{(2)} = 2 \quad \Rightarrow B$ pos. def. nach Hauptminorenkrit.

$A = \cancel{A} \quad \cdot -\frac{4}{3}$

$$\begin{matrix} 3 & 4 \\ 4 & 5 \end{matrix} \rightarrow \begin{matrix} 3 & 0 \\ 4 & -\frac{16}{3}+5 \end{matrix} \; \cdot -\frac{4}{3} \rightarrow \begin{matrix} 3 & 0 \\ 0 & -\frac{1}{3} \end{matrix} \Rightarrow \text{indefinit}$$

b) fasse B als SKP-Matrix $\iota(E,E)$ auf

$$\sigma(E,E) = \langle E^*, f(E)\rangle \cdot \iota(E,E)$$

$$\Leftrightarrow \langle E^*, f(E)\rangle = \left(\iota(E,E)\right)^{-1} \cdot \sigma(E,E)$$

$$\Leftrightarrow F := B^{-1} \cdot A = \frac{1}{2}\begin{pmatrix} 11 & -8 \\ -8 & 6 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 2 \\ 0 & -1 \end{pmatrix}$$

$$\chi_F(x) = \begin{vmatrix} \frac{1}{2}-x & 2 \\ 0 & -1-x \end{vmatrix} = -\left(\frac{1}{2}-x\right)(1+x)$$

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = -1$$

EV: zu λ_1

$$\begin{matrix} 0 & 2 \\ 0 & -\frac{3}{2} \end{matrix} \Rightarrow p_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\cancel{\Rightarrow P = \begin{pmatrix} 1 & 4 \\ 0 & -3 \end{pmatrix}}$$

zu λ_2

$$\begin{matrix} \frac{3}{2} & 2 \\ 0 & 0 \end{matrix} \Rightarrow p_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$\cancel{\|p_2\| = 5} \Rightarrow \cancel{P =} \quad P = \frac{1}{5}\begin{pmatrix} 5 & 4 \\ 0 & -3 \end{pmatrix}$$

$$\cancel{\Rightarrow P^T A P = \operatorname{diag}\left(\tfrac{1}{2}, -1\right) \qquad P^T \cdot B \cdot P = E_2}$$

Aufgabe 16

$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \in \mathbb{R}^{2\times 3} \qquad \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

a) $\operatorname{rg} A = 2 \Rightarrow A^+ = A^T \cdot (A \cdot A^T)^{-1}$

$= \frac{1}{3} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}$

$A \cdot A^T = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$\Rightarrow (A \cdot A)^T = \frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

b) $A \cdot (x_j) = (s_i)$

$\Leftrightarrow \quad (x_j) = A^+ \cdot (s_i)$

$\Rightarrow \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \frac{3}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

Aufgabe 17

$\operatorname{rg} A = 3$

$$\Rightarrow A^+ = \cancel{A^T}\,(A^T \cdot A)^{-1} \cdot A^T$$

$$= \frac{1}{10}\begin{pmatrix} 9 & 2 & -4 \\ 2 & 6 & -2 \\ -4 & -2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 9 & -2 & 1 & 2 \\ 2 & 4 & -2 & 6 \\ -4 & 2 & 4 & -2 \end{pmatrix}$$

$$A^T \cdot A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 5 \end{pmatrix}$$

$$\begin{array}{ccc} & & \cdot -1 \\ 2 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 5 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \quad\Bigg|\quad \begin{array}{ccc} & \cdot -2 & \\ 2 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 3 \\ \hline 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \quad\Bigg|\quad \begin{array}{ccc} \cdot \frac{1}{2} & \cdot -\frac{1}{5} & \\ 2 & 0 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 3 \\ \hline 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{array}$$

$$\begin{array}{ccc} & \cdot -1 & \cdot -3 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 3 \\ \hline \frac{1}{2} & -\frac{2}{5} & -1 \\ 0 & -\frac{1}{5} & 0 \\ 0 & \frac{2}{5} & 1 \end{array} \quad\Bigg|\quad \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline \frac{9}{10} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & +\frac{3}{5} & -\frac{1}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{2}{5} \end{array} = \frac{1}{10}\begin{pmatrix} 9 & 2 & -4 \\ 2 & 6 & -2 \\ -4 & -2 & 4 \end{pmatrix}$$

$$(x_i) = A^+ \cdot (s_i) = \frac{1}{10}\begin{pmatrix} 9 & -2 & 1 & 2 \\ 2 & 4 & -2 & 6 \\ -4 & 2 & 4 & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 9 \\ 0 \\ -4 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 10 \\ \cancel{10}\,20 \\ 10 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Aufgabe 19 $\quad (\mathbb{R}^{4\times 1}, \cdot)$ eukl.

$$a_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \quad a_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

a)
$$G(a_1, a_2) = \begin{pmatrix} a_1 \cdot a_1 & a_1 \cdot a_2 \\ a_2 \cdot a_1 & a_2 \cdot a_2 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 15 \end{pmatrix}$$

$$\det G(a_1, a_2) = 45 - 4 = 41$$

b) $\sqrt{41}$

c)
$$\cos \sphericalangle (a_1, a_2) = \frac{a_1 \cdot a_2}{\|a_1\| \, \|a_2\|} = \frac{2}{3 \cdot 15} = \frac{2}{45}$$

d) nein in $\mathbb{R}^{4\times 1}$ ~~bestimme~~ hat nur ein Tripel von Vektoren ein Vektorprodukt