26.3.09

Wahrscheinlichkeitsrechnung u. Statistik UE

II, 8,

a) $P(\text{Sieg}) = P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3^c) + P(A_1^c \cap A_2 \cap A_3)$ mit $A_i :=$ Sieg in der i. Partie

$= \text{———} \;\text{"}\; \text{———} + 2P(A_1 \cap A_2 \cap A_3^c)$

$$\Rightarrow P(CSC) = p_c p_s p_c + 2 p_c p_s (1-p_c)$$
$$= p_c^2 p_s + 2 p_c p_s - 2 p_c^2 p_s$$
$$= p_c p_s (2-p_c)$$
$$\Rightarrow P(SCS) = p_s p_c (2-p_s) \qquad \text{(analog)}$$

VS: $p_c < p_s$
$\Leftrightarrow 2-p_c > 2-p_s$
$\Leftrightarrow p_c p_s (2-p_c) > p_c p_s (2-p_s) \quad \Rightarrow \quad P(CSC) > P(SCS)$

b) $P(\text{Sieg}) = P(A_1 \cap A_2 \cap A_3) + 2P(A_1 \cap A_2 \cap A_3^c) + P(A_1 \cap A_2^c \cap A_3)$

Beh.: $P(CSC) < P(SCS)$

$\Leftrightarrow p_s p_c (2-p_c) + p_c^2(1-p_s) < p_s p_c (2-p_s) + p_s^2(1-p_c)$

$\Leftrightarrow \cancel{2p_s p_c} - p_c^2 p_s + p_c^2 - p_c^2 p_s < \cancel{2 p_s p_c} - p_s^2 p_c + p_s^2 - p_s^2 p_c$

$\Leftrightarrow 2 p_s p_c \cancel{(p_s - p_c)} < p_s^2 - p_c^2 = (p_s + p_c)\cancel{(p_s - p_c)}$

$\Leftrightarrow 2 p_s p_c < 2 p_c < p_s + p_c$ ✓

9, zz: $|P(A) - P(B)| \leq P(A \triangle B)$

$$P(A \triangle B) = P((A \setminus (A\cap B)) \cup (B \setminus (A \cap B)))$$
$$= P(A \setminus (A \cap B)) + P(B \setminus (A \cap B)) - P(\overbrace{(A\setminus(A\cap B)) \cap (B \setminus (A \cap B))}^{\emptyset})$$
$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

•) $P(A) > P(B) \quad \Rightarrow \quad \cancel{P(A)} - P(B) \leq \cancel{P(A)} + P(B) - 2P(A \cap B)$
$\Leftrightarrow P(B) \leq P(A \cap B)$ ✓

•) $P(B) > P(A)$ analog

10) Ann.: $P(M_i | B) \le P(M_i) \quad \forall i \in \{2, 3, \dots, k\}$

$$1 = P(\Omega | B) = \sum_{i=1}^{k} P(M_i | B) < \sum_{i=1}^{k} P(M_i) = 1 \quad ↯$$

11)

$$n\, 2^{n-1} = n \cdot \sum_{j=0}^{n-1} \binom{n-1}{j} = \sum_{j=0}^{n-1} \frac{n(n-1)!}{(n-1-j)!\, j!} = \sum_{j=0}^{n-1} (n-j) \frac{n!}{(n-j)!\, j!}$$

$$= \sum_{j=0}^{n} (n-j) \binom{n}{j} = \sum_{j=0}^{n} j \binom{n}{j}$$

12)

$a + b < 1$

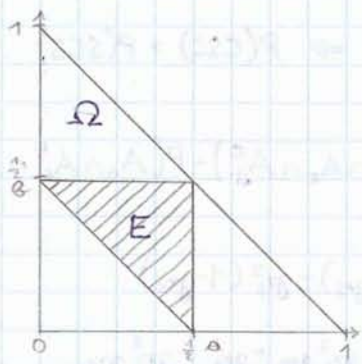
$a, b > 0$

$\Omega = \{(a,b) \in \mathbb{R} \times \mathbb{R} \mid a + b < 1 \wedge a, b > 0\}$

$\Rightarrow A(\Omega) = \frac{1}{2}$

Dreieck möglich $\Leftrightarrow a + b > 1 - a - b \Leftrightarrow a + b > \frac{1}{2}$

$=: E$ $\qquad a, b < \frac{1}{2}$



$$P(E) = \frac{A(E)}{A(\Omega)} = \frac{1}{8} \cdot 2 = \frac{1}{4}$$

13) $\Omega = \{L, R\} \qquad P(L) = \frac{1}{2}$

unabh. Ereignisse $\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

$k = 20$: $P(L \cap L \cap \dots \cap L) = P(L)^{20} = \left(\frac{1}{2}\right)^{20}$

$k = 19$: $P(R \cap L \cap \dots \cap L) + P(L \cap R \cap \dots \cap L) + P(L \cap L \cap \dots \cap R) = 20\, P(L)^{20} P(R) = 20 \left(\frac{1}{2}\right)^{21}$

$\Rightarrow$ allgemein: $\binom{40-k}{20} \left(\frac{1}{2}\right)^{40-k}$

Anz. Mögl. für 20 Griffe nach links

14) a) ZZ: $\mathcal{M}$ ist σ-Algebra

- $\Omega \in \mathcal{M}$ ✓
- $A \in \mathcal{M} \Rightarrow A^c \in \mathcal{M}$ ✓
- $\bigcup A_i \in \mathcal{M}$ ✓

b) $\mathcal{Z} = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4, \omega_6\}, \{\omega_5\}, \emptyset, \Omega\}$