20.4.09

Analysis UE

V, 45, 88, 92, 96, 100, 103, 125

45) $\displaystyle\int \frac{dx}{a \sinh x + b \cosh x} = \int \frac{dx}{A \cosh\alpha \sinh x + A \sinh\alpha \cosh x}$

<u>Warum geht das?</u>

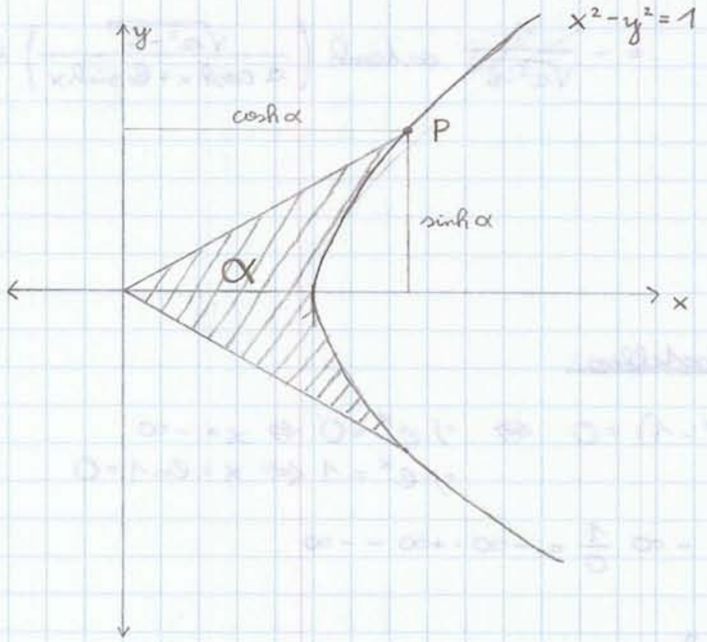
Seien $(a, b) \in \mathbb{R}^2$ vorgegeben (Punkt in der Ebene).

(a, b) erfüllt die Hyperbel-Gleichung: $\frac{1}{a^2-b^2}(x^2-y^2) = 1$

$$\Leftrightarrow \left(\frac{x^2}{a^2-b^2} - \frac{y^2}{a^2-b^2}\right) = 1$$

$$\Leftrightarrow \left(\frac{x}{\sqrt{a^2-b^2}}\right)^2 - \left(\frac{y}{\sqrt{a^2-b^2}}\right)^2 = 1$$

$\Rightarrow P := \Big(\underbrace{\frac{a}{\sqrt{a^2-b^2}}}_{\cosh\alpha}, \underbrace{\frac{b}{\sqrt{a^2-b^2}}}_{\sinh\alpha}\Big)$ erfüllt $x^2-y^2=1$. (Einheitshyperbel)



$\Rightarrow A = \sqrt{a^2-b^2}$. Jeder Punkt im Raum bestimmt in eindeutiger Weise einen Punkt auf der Einheitshyperbel.

$$\int \frac{dx}{a \sinh x + b \cosh x} = \int \frac{dx}{A \cosh\alpha \sinh x + A \sinh\alpha \cosh x}$$

$$= \frac{1}{A} \int \frac{dx}{\cosh\alpha \sinh x + \sinh\alpha \cosh x}$$

$$= \frac{1}{A} \int \frac{dx}{\sinh(x+\alpha)}$$

$$= \left| \begin{matrix} x+\alpha = y \\ dx = dy \end{matrix} \right| = \frac{1}{A} \int \frac{dy}{\sinh y}$$

$$= \left| \begin{matrix} \tanh y = t \\ (1-t^2)\, dy = dt \end{matrix} \right| = \frac{1}{A} \int \frac{\sqrt{1-t^2}}{t\,(1-t^2)}\, dt = \frac{1}{A} \int \frac{dt}{t\sqrt{1-t^2}}$$

$$= \left| \begin{matrix} \sqrt{1-t^2} = u \\ \frac{-\cancel{2}t}{\cancel{2}u}\, dt = du \end{matrix} \right| = -\frac{1}{A} \int \frac{\cancel{t}\, du}{\sqrt{u^2-1}\, \cancel{t} \sqrt{u^2-1}} = -\frac{1}{A} \int \frac{du}{u^2-1}$$

$|u| = |\sqrt{1-\tanh^2 y}| = \left|\frac{1}{\cosh y}\right| < 1 \ \forall y \in \mathbb{R}\setminus\{0\}$

$$= -\frac{1}{A} \operatorname{artanh} u + c = -\frac{1}{A} \operatorname{artanh} \frac{1}{\cosh(x+\alpha)} + c$$

$$= -\frac{1}{A} \operatorname{artanh}\left(\frac{1}{\cosh x \underbrace{\cosh\alpha}_{\frac{a}{A}} + \sinh x \underbrace{\sinh\alpha}_{\frac{b}{A}}} \right) + c$$

$$= -\frac{1}{\sqrt{a^2-b^2}} \operatorname{artanh}\left(\frac{\sqrt{a^2-b^2}}{a \cosh x + b \sinh x} \right) + c$$

88) $\displaystyle\int_0^1 \frac{\ln x}{e^{2x}-e^x}\, dx$

Uneigentlichkeitsstellen:

$$e^{2x}-e^x = e^x(e^x-1) = 0 \iff \begin{matrix} \bullet)\ e^x = 0 \iff x = -\infty \\ \bullet)\ e^x = 1 \iff x = \ln 1 = 0 \end{matrix}$$

$$\lim_{x\to 0^+} \frac{\ln x}{\underbrace{e^{2x}-e^x}_{\substack{>0 \text{ für } x>0 \\ \text{da } e^x \text{ monoton } \nearrow}}} = -\infty \, \frac{1}{0} = -\infty \cdot +\infty = -\infty$$

$\Rightarrow \int$ uneigentlich an $x = 0$

Konvergenz:

$$\ln x \le 0 \wedge e^{2x}-e^x \ge 0 \ \forall x \in [0,1] \Rightarrow \frac{\ln x}{e^{2x}-e^x} \le 0 \ \forall x \in [0,1]$$

$$\Rightarrow \int_0^1 \frac{\ln x}{e^{2x}-e^x}\, dx = -\int_0^1 \left| \frac{\ln x}{e^{2x}-e^x} \right| dx \le 0$$

$$\left|\frac{\ln x}{e^{2x}-e^x}\right| = \frac{|\ln x|}{e^{2x}-e^x} \geq \frac{1}{e^{2x}-e^x} \iff |\ln x| \geq 1$$
$$\iff \ln x \leq -1$$
$$\iff x \leq e^{-1}$$

$$\Rightarrow \int_0^{e^{-1}} \left|\frac{\ln x}{e^{2x}-e^x}\right| dx \geq \int_0^{e^{-1}} \frac{dx}{e^x(e^x-1)} = \left| \begin{matrix} e^x = t \\ t\,dx = dt \end{matrix} \right|$$

$$= \int_1^{e^{e^{-1}}} \frac{dt}{t^2(t-1)} = \int_1^{e^{e^{-1}}} \left(-\frac{1}{t^2} - \frac{1}{t} + \frac{1}{t-1}\, dt\right)$$

$$= \lim_{\alpha \to 1^+} \left(\frac{1}{t} - \ln|t| + \ln|t-1|\right)\Big|_\alpha^{e^{e^{-1}}}$$

$$= \frac{1}{e^{e^{-1}}} - e^{-1} + \ln|e^{e^{-1}}-1| - 1 - 0 + \lim_{\alpha \to 1^+} \ln|\alpha - 1| = -\infty$$

$$\Rightarrow \int_0^1 \frac{\ln x}{e^{2x}-e^x} = \int_0^{e^{-1}} \frac{\ln x}{e^{2x}-e^x} + \underbrace{\int_{e^{-1}}^1 \frac{\ln x}{e^{2x}-e^x}}_{\text{Riemann-Int.}} = -\infty, \text{ also divergent.}$$

92) $\displaystyle\int_0^\infty \frac{(x-x^3)\ln^2 x}{(1+x^2)^3}\, dx =: \int_0^\infty f(x)\, dx$

<u>Uneigentlichkeitsstellen:</u>

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{1-x^2}{(1+x^2)^3} \cdot \lim_{x \to 0^+} x \ln^2 x$$

$$= \lim_{x \to 0^+} \frac{\ln^2 x}{\frac{1}{x}} \overset{\text{de l'H}}{=} \lim_{x \to 0^+} \frac{2\ln x \cancel{\frac{1}{x}}}{-\frac{1}{x^{\cancel{2}}}} \overset{\text{de l'H}}{=} \lim_{x \to 0^+} \frac{2\frac{1}{x}}{\frac{1}{x^2}} = \lim_{x \to 0^+} 2x = 0$$

$\Rightarrow \int$ uneigtl. an ∞

<u>Konvergenz:</u>

$\displaystyle\int_0^1 f(x)\, dx$ ist Riemann-Int.

$$\int_1^\infty f(x)\, dx = \int_1^\infty \underbrace{(x-x^3)\ln^2 x}_{=g(x)} \underbrace{\frac{1}{(1+x^2)^3}}_{=h(x)} dx$$

$$\int_1^x \underbrace{(t-t^3)}_{f'} \underbrace{\ln^2 t}_{g}\, dt = \left(\frac{t^2}{2} - \frac{t^4}{4}\right)\ln^2 t \Big|_1^x - \int_1^x \Big(\underbrace{\frac{t^2}{\cancel{2}} - \frac{t^4}{4\cancel{2}}}_{f'}\Big) \cancel{2}\, \underbrace{\ln t}_{g}\, \cancel{\tfrac{1}{t}}\, dt$$

$$= \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\ln^2 x - \left(\frac{t^2}{2} - \frac{t^4}{8}\right)\ln t \Big|_1^x + \underbrace{\int_1^x \left(\frac{t^{\cancel{2}}}{2} - \frac{t^{\cancel{4}3}}{8}\right)\cancel{\tfrac{1}{t}}\, dt}_{\frac{t^2}{4} - \frac{t^4}{32}\Big|_1^x}$$

$$= \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\ln^2 x - \left(\frac{x^2}{2} - \frac{x^4}{8}\right)\ln x + \frac{x^2}{4} - \frac{x^4}{32} - \frac{1}{4} + \frac{1}{32}$$

$D\left(\int_1^x g\right) = \mathbb{R} \Rightarrow \int_1^x g$ beschr. auf $[1,\infty)$.

$f(x) \le 0 \quad \forall x \ge 1$ da $1-x^2 \le 0$

$$\Rightarrow \left| \frac{x(1-x^2)\ln^2 x}{(1+x^2)^3} \right| = \frac{x(x^2-1)\ln^2 x}{(1+x^2)^3}$$

$$\le \frac{x\cancel{(x^2+1)}\ln^2 x}{(1+x^2)^{\cancel{3}\,2}}$$

$\ln^2 x \le x$

$\Leftrightarrow \ln x \le \sqrt{x}$

$x=1$: $\ln 1 = 0 \le 1$

$x>1$: vgl. Steigung

$\frac{1}{x} \le \frac{2}{\sqrt{x}}$

$$\le \frac{x^2}{(1+x^2)^2} = \frac{x^2}{1+2x^2+x^4}$$

$$= \frac{1}{\frac{1}{x^2}+2+x^2} \le \frac{1}{x^2} \qquad \forall x \ge 1$$

$$\int_1^\infty |f(x)|\,dx \le \int_1^\infty \frac{dx}{x^2} = -\frac{1}{x}\Big|_1^\infty = 0+1 = 1$$

also $\int_0^\infty f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^\infty f(x)\,dx$ konvergent.

96) $\displaystyle\int_1^\infty \frac{(x-x^3)\ln^2 x}{(1+x^2)^3}\,dx = \left|\begin{matrix} f'(x)=\frac{(x-x^3)}{(1+x^2)^3} \\ g(x)=\ln^2 x\end{matrix}\right|$

$$\int \frac{x(1-x^2)}{(1+x^2)^3}\,dx = \left|\begin{matrix}1+x^2=t\\ 2x\,dx=dt\end{matrix}\right| = \frac12\int\frac{2-t}{t^3}\,dt = \int\frac{1}{t^3}\,dt-\frac12\int\frac{1}{t^2}\,dt$$

$$= -\frac{1}{2t^2}+\frac{1}{2t}+c = \frac12\left(-\frac{1}{(1+x^2)^2}+\frac{1}{1+x^2}\right)+c = \frac{x^2}{2(1+x^2)^2}+c$$

$$= \lim_{\beta\to\infty}\left(\frac{x^2\ln^2 x}{2(1+x^2)^2}\Bigg|_1^\beta - \int_1^\beta \underbrace{\frac{x}{(1+x^2)^2}}_{f'(x)}\,\underbrace{\ln x}_{g(x)}\,dx\right)$$

$$\int \frac{x}{(1+x^2)^2}\,dx = \left|\begin{matrix}1+x^2=t\\ 2x\,dx=dt\end{matrix}\right| = \frac12\int\frac{dt}{t^2} = -\frac{1}{2t}+c = -\frac{1}{2(1+x^2)}+c$$

$$= \lim_{x\to\infty}\frac{x^2\ln^2 x}{2(1+x^2)^2} - \lim_{\beta\to\infty}\left(-\frac{\ln x}{2(1+x^2)}\Bigg|_1^\beta + \frac12\int_1^\beta\frac{dx}{x(1+x^2)}\right)$$

$$\int\frac{dx}{x(1+x^2)} = \left|\begin{matrix}1+x^2=t\\ 2x\,dx=dt\end{matrix}\right| = \frac12\int\frac{dt}{(t-1)t} = \frac12\left(\int\frac{dt}{t-1}+\int\frac{dt}{t}\right)$$

$$= \frac12\left(\ln(t-1)-\ln(t)\right)+c = \frac12\left(\ln(x^2)-\ln(1+x^2)\right)+c$$

$$= \lim_{x\to\infty}\frac{x^2\ln^2 x}{2(1+x^2)^2} + \lim_{x\to\infty}\frac{\ln x}{2(1+x^2)} - \lim_{\beta\to\infty}\frac14\left(\ln(x^2)-\ln(1+x^2)\right)\Bigg|_1^\beta$$

$$= \text{''} \qquad \text{''} \qquad \lim_{x\to\infty} \text{''} \qquad - \frac{\ln 2}{4}$$

$$\lim_{x\to\infty}\frac{x^2\ln^2 x}{(1+x^2)^2} = \left(\lim_{x\to\infty}\frac{x\ln x}{1+x^2}\right)^2 \overset{\text{de l'H}}{=} \left(\lim_{x\to\infty}\frac{\ln x+1}{2x}\right)^2 \overset{\text{de l'H}}{=} \left(\lim_{x\to\infty}\frac{\frac1x}{2}\right)^2 = 0$$

$\displaystyle\lim_{x\to\infty}\frac{\ln x}{1+x^2} = 0$ (analog)

$$\lim_{x\to\infty}\left(\ln x^2-\ln(1+x^2)\right) = \lim_{x\to\infty}\ln\left(\frac{x^2}{1+x^2}\right) = \ln\left(\lim_{x\to\infty}\frac{x^2}{1+x^2}\right) = \ln 1 = 0$$

$$\Rightarrow \int_1^\infty\frac{(x-x^3)\ln^2 x}{(1+x^2)^3}\,dx = -\frac{\ln 2}{4}$$

$$\int_0^1 \frac{(x-x^3)\ln^2 x}{(1+x^2)^3}\,dx = \lim_{\alpha\to 0^+}\left(\frac{x^2\ln^2 x}{2(1+x^2)^2}\Big|_\alpha^1 + \frac{\ln x}{2(1+x^2)}\Big|_\alpha^1 - \frac{1}{4}\left(\ln x^2-\ln(1+x^2)\right)\Big|_\alpha^1\right)$$

$$= -\lim_{x\to 0^+}\frac{x^2\ln^2 x}{2(1+x^2)^2} - \lim_{x\to 0^+}\frac{\ln x}{2(1+x^2)} + \frac{\ln 2}{4} + \lim_{x\to 0^+}\frac{1}{4}\left(\ln x^2-\ln(1+x^2)\right)$$

$$\lim_{x\to 0^+}\frac{x^2\ln^2 x}{(1+x^2)^2} = \lim_{x\to 0^+} x^2\ln^2 x = \left(\lim_{x\to 0^+} x\ln x\right)^2 = \left(\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}}\right)^2 \overset{\text{de l'H}}{=} \left(\lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}\right)^2 = 0$$

$$\lim_{x\to 0^+}\left(\frac{1}{\cancel{4}_2}\big(\overbrace{\ln x^2}^{2\ln x} - \overbrace{\ln(1+x^2)}^{0}\big) - \frac{\ln x}{\underbrace{2(1+x^2)}_{1}}\right) = 0$$

$$\Rightarrow \int_0^1 \frac{(x-x^3)\ln^2 x}{(1+x^2)^3}\,dx = \frac{\ln 2}{4}$$

also $\displaystyle\int_0^\infty \frac{(x-x^3)\ln^2 x}{(1+x^2)^3} = 0.$

100) $\displaystyle\int_0^2 \underbrace{\frac{6x^2+9x+11}{x^3+2x^2-x-2}}_{=:f(x)}\,dx$

Uneigentlichkeitsstellen:

$D(f) = \mathbb{R}\setminus\{NS \text{ des Nenners}\}$

$x^3+2x^2-x-2=0$

$\Leftrightarrow x(x^2-1)+2(x^2-1)=0$

$\Leftrightarrow (x+1)(x-1)(x+2)=0 \quad \Rightarrow NS: \{-2,-1,1\}$

also $\int_0^2 f$ uneigtl. an $x=1$

Konvergenz:

$$\lim_{\beta\to 1^-}\int_0^\beta \frac{6x^2+9x+11}{(x+1)(x-1)(x+2)}\,dx$$

$$6x^2+9x+11 = A(x-1)(x+2)+B(x+1)(x+2)+C(x^2-1)$$

$$\left.\begin{array}{l} x^2:\ 6 = A+B+C \\ x:\ 9 = A+3B \\ x^0:\ 11 = -2A+2B-C \end{array}\right\} \Rightarrow B=\frac{13}{3},\ A=-4,\ C=\frac{17}{3}$$

$$\Rightarrow \int \frac{6x^2+9x+11}{x^3+2x^2-x-2}\,dx = -4\int\frac{dx}{x+1} + \frac{13}{3}\int\frac{dx}{x-1} + \frac{17}{3}\int\frac{1}{x+2}$$

$$= -4\ln|x+1| + \frac{13}{3}\ln|x-1| + \frac{17}{3}\ln|x+2| + c$$

$$\left.\begin{array}{l} \displaystyle\lim_{\beta\to 1^-}\int_0^\beta f = -4\ln 2 + \frac{13}{3}\ln 0 + \frac{17}{3}\ln 3 + 4\ln 1 - \frac{13}{3}\ln 1 - \frac{17}{3}\ln 2 = -\infty \\ \displaystyle\lim_{+}\int^2 f = +\infty \text{ analog} \end{array}\right\} \Rightarrow \int_0^2 f \text{ divergent}$$

Cauchyscher Hauptwert:

$$CH\int_0^2 f = \lim_{\varepsilon\to 0^+}\left(\int_0^{1-\varepsilon} f + \int_{1+\varepsilon}^2 f\right)$$

$$= \lim_{\varepsilon\to 0^+}\Big(\cancel{-4\ln(2-\varepsilon}\; \cancel{-4\ln 2} + \frac{13}{3}\ln|-\varepsilon| + \cancel{\frac{17}{3}\ln 3} - \frac{17}{3}\ln 2$$

$$-4\ln 3 + \frac{17}{3}\ln 4 + \cancel{4\ln 2} - \frac{13}{3}\ln\varepsilon - \cancel{\frac{17}{3}\ln 3}$$

$$= \frac{17}{3}(\overbrace{\ln 4 - \ln 2}^{\ln 2}) - 4\ln 3 + \frac{13}{3}\underbrace{\lim_{\varepsilon\to 0^+}(\ln\varepsilon - \ln\varepsilon)}_{=0}$$

$$= \frac{17}{3}(\ln 2) - 4\ln 3$$

103)

$$\int \underbrace{\sin bx}_{g(x)}\, \underbrace{e^{-ax}}_{f'(x)}\,dx = -\frac{1}{a}\sin bx\, e^{-ax} + c + \frac{b}{a}\int \underbrace{\cos bx}_{g(x)}\,\underbrace{e^{-ax}}_{f'(x)}\,dx$$

$$= \text{———}\;\text{"}\;\text{———} - \frac{b}{a^2}\cos bx\, e^{-ax} - \frac{b^2}{a^2}\int \sin bx\, e^{-ax}\,dx$$

$$\Leftrightarrow \overbrace{\left(1+\frac{b^2}{a^2}\right)}^{\frac{a^2+b^2}{a^2}}\int \sin bx\, e^{-ax}\,dx = -\frac{1}{a}\sin bx\, e^{-ax} - \frac{b}{a^2}\cos bx\, e^{-ax} + c$$

$$\Leftrightarrow \int \sin bx\, e^{-ax}\,dx = -\frac{a^{\cancel{2}}}{\cancel{a}(a^2+b^2)}\sin bx\, e^{-ax} - \frac{\cancel{a^2}b}{\cancel{a^2}(a^2+b^2)}\cos bx\, e^{-ax} + c$$

$$= \frac{-1}{a^2+b^2}e^{-ax}(a\sin bx + b\cos bx) + c$$

$$\int_0^\infty \sin bx\, e^{-ax}\,dx = \lim_{x\to\infty}\left(-\frac{1}{a^2+b^2}e^{-ax}(a\sin bx + b\cos bx)\right) + \frac{b}{a^2+b^2}$$

$a=0$: $\lim_{x\to\infty}\left(-\frac{1}{b^{\cancel{2}}}\cancel{e^0}\,\cancel{b}\cos bx\right) = \lim_{x\to\infty} -\frac{\cos bx}{b}$ divergiert für $x\to\infty$

$a \neq 0$:

$$\lim_{x\to\infty}\left(-\frac{1}{a^2+b^2}e^{\mathrm{sgn}(a)(-\infty)}\underbrace{(a\sin bx + b\cos bx)}_{\text{beschr.}}\right) = \begin{cases} a>0 & 0 \\ a<0 & \notin\mathbb{R}\end{cases}$$

$$\Rightarrow \int_0^\infty e^{-ax}\sin bx\,(\exists \text{ für } a>0) = \frac{b}{a^2+b^2}$$

125) $d\left(\binom{a}{b},\binom{c}{d}\right)=\left|\frac{1}{a}-\frac{1}{c}\right|+\left|b^3-d^3\right|$

ZZ: d ist Metrik auf $(\mathbb{R}^+)^2$

<u>0. nichtnegativ</u>

$$d\left(\binom{a}{b},\binom{c}{d}\right)\geq 0 \quad \forall \left(\binom{a}{b},\binom{c}{d}\right)\in(\mathbb{R}^+)^2\times(\mathbb{R}^+)^2 \quad ✓$$

<u>1. $d(x,y)=0 \Leftrightarrow x=y$</u>

$$d\left(\binom{a}{b},\binom{c}{d}\right)=\underbrace{\left|\frac{1}{a}-\frac{1}{c}\right|}_{\geq 0}+\underbrace{\left|b^3-d^3\right|}_{\geq 0}=0$$

$$\Leftrightarrow \left|\frac{1}{a}-\frac{1}{c}\right|=\left|b^3-d^3\right|=0 \Leftrightarrow a=c \wedge b=d \Leftrightarrow \binom{a}{b}=\binom{c}{d} \quad ✓$$

<u>2. Symmetrie</u>

$$d\left(\binom{a}{b},\binom{c}{d}\right)=\left|\frac{1}{a}-\frac{1}{c}\right|+\left|b^3-d^3\right|=\left|\frac{1}{c}-\frac{1}{a}\right|+\left|d^3-b^3\right|=d\left(\binom{c}{d},\binom{a}{b}\right) \quad ✓$$

<u>3. Dreiecksungleichung</u>

$$d\left(\binom{a}{b},\binom{c}{d}\right)+d\left(\binom{c}{d},\binom{e}{f}\right)=\left|\frac{1}{a}-\frac{1}{c}\right|+\left|b^3-d^3\right|+\left|\frac{1}{c}-\frac{1}{e}\right|+\left|d^3-f^3\right|$$

$$=\underbrace{\left|\frac{1}{a}-\frac{1}{c}\right|+\left|\frac{1}{c}-\frac{1}{e}\right|}_{\geq \left|\frac{1}{a}-\frac{1}{e}\right|}+\underbrace{\left|b^3-d^3\right|+\left|d^3-f^3\right|}_{\geq \left|b^3-f^3\right|}$$

$$\geq \left|\frac{1}{a}-\frac{1}{e}\right|+\left|b^3-f^3\right|=d\left(\binom{a}{b},\binom{e}{f}\right) \quad ✓$$

$K_1\binom{1}{1}$ bestimmen und skizzieren.

$$K_1\binom{1}{1}=\left\{\binom{x}{y}\in(\mathbb{R}^+)^2 \;\middle|\; d\left(\binom{1}{1},\binom{x}{y}\right)<1\right\}$$

$$=\left\{\binom{x}{y}\in(\mathbb{R}^+)^2 \;\middle|\; \left|1-\frac{1}{x}\right|+\cancel{\left|1-\frac{1}{y}\right|}\,|1-y^3|<1\right\}$$

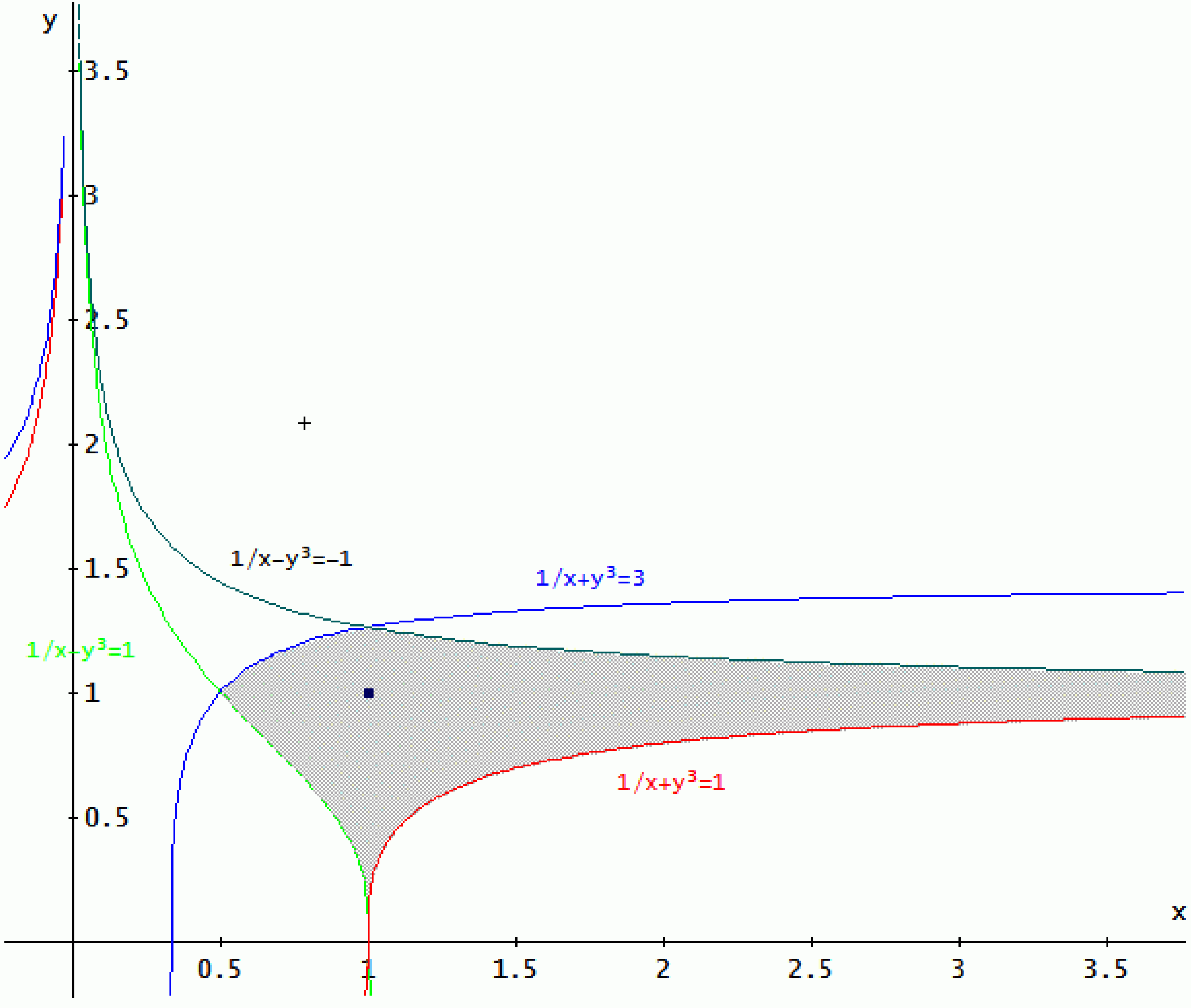
~~Fall:~~ <u>$x,y\geq 1$:</u> $\cancel{1-\frac{1}{x}+1-\frac{1}{y^3}<1 \Leftrightarrow \frac{1}{x}+\frac{1}{y^3}>1}$ $\quad$ $\cancel{1}-\frac{1}{x}+y^3-\cancel{1}<1 \Leftrightarrow y^3-\frac{1}{x}<1$

<u>$x,y\leq 1$:</u> $\cancel{\frac{1}{x}-1+\frac{1}{y^3}-1<1 \Leftrightarrow \frac{1}{x}+\frac{1}{y^3}<3}$ $\quad$ $\frac{1}{x}-\cancel{1}+\cancel{1}-y^3<1 \Leftrightarrow \frac{1}{x}-y^3<1$

<u>$x\geq 1, y\leq 1$:</u> $\cancel{1-\frac{1}{x}+\frac{1}{y^3}-1<1 \Leftrightarrow \frac{1}{y^3}-\frac{1}{x}<1}$ $\quad$ $1-\frac{1}{x}+1-y^3<1 \Leftrightarrow \frac{1}{x}+y^3>1$

<u>$x\leq 1, y\geq 1$:</u> $\cancel{\frac{1}{x}-1+1-\frac{1}{y^3}<1 \Leftrightarrow \frac{1}{x}-\frac{1}{y^3}<1}$ $\quad$ $\frac{1}{x}-1+y^3-1<1 \Leftrightarrow \frac{1}{x}+y^3<3$

$$\Rightarrow K_1\binom{1}{1}=\left\{\binom{x}{y}\in(\mathbb{R}^+)^2 \;\middle|\; 1<\frac{1}{x}+\cancel{\frac{1}{y^3}}\,y^3<3 \wedge -1<\frac{1}{x}-\cancel{\frac{1}{y^3}}\,y^3<1\right\}$$


y
3.5
3
2.5
2
1.5
1
0.5
x
0.5
1
1.5
2
2.5
3
3.5
1/x−y³=−1
1/x+y³=3
1/x−y³=1
1/x+y³=1